# **MATHEMATICS (US)**

Paper 0444/11

Paper 1

# Key messages

To succeed in this paper, candidates need to have completed the full Core syllabus, be able to remember and apply formulae and to give answers in the form required. Candidates are reminded of the need to read the question carefully, focussing on key words or instructions.

# **General comments**

Candidates must check their work for sense and accuracy as there were answers in context that made little sense, for example in **Questions 1**, **10** and **18**. Candidates must show all working to enable method marks to be awarded. This is vital in 2 or multi-step problems, in particular with algebra, where each step should be shown separately to maximise the chance of gaining marks in, for example, **Questions 11**, **13**, **16**, and **20**. Candidates must take note of the form or units that are required, for example, in **Questions 6** and **10**.

The questions that presented least difficulty were **Questions 6(b)**, 7, 12, 17, 18(c)(ii) and 21(a)(i). Those that proved to be the most challenging were **Questions 1**, 8(b), 15, 19(b), some of 21, and 22(c) and these along with **Question 18** were the ones that were most likely to be left blank. It is likely that the blank responses were due to the syllabus areas being tested rather than lack of time.

# **Comments on specific questions**

# **Question 1**

This question on time was reasonably straightforward. What made this question on time slightly more challenging was that the period goes over midnight. Workings were useful here for candidates to check they had the correct time for each day. Candidates got confused with the number of minutes so 8 rather than 52 was seen frequently. There were some answers that had a number of hours and more than 59 minutes.

Answer: 8 (h) 52 (min)

## **Question 2**

The most common incorrect answer came from candidates dividing 25 by 3 instead of using the correct method. Others gave 75 from 25 multiplied by 3. There were also answers of 22 (25 - 3) or 28 (25 + 3).

Answer. 12

#### Question 3

The form of the probability was not specified so candidates could give a fraction or percentage instead of a decimal if they wished. If candidates choose to give a percentage, they must include the percent-sign. Candidates did well here but some gave fractions based on 28 rather than working out 1 - 0.28.

Answer. 0.72

# **Question 4**

The most frequent error was for candidates to give 127 000. Often, those that realised that this was a number less than 1, made errors in the number of zeros to include.



Answer: 0.00127

## **Question 5**

Some candidates gave an answer numerically larger than 60 000. Candidates need to know that if you change from meters to larger units, kilometers, the actual number decreases.

Answer: 60

#### **Question 6**

In **part (a)**, candidates gave 7 or 0 as their answer instead of 1. Candidates were more successful in **part (b)** with many more reaching the correct answer. A few gave 7<sup>5</sup> which did not score the mark as only the power was asked for. A small number tried to work out 7<sup>5</sup> which didn't score the mark.

Answers: (a) 1 (b) 5

## **Question 7**

Many candidates were successful in this question. For **part (a)**, many were able to answer correctly with incorrect answers such as isosceles and hypotenuse seen. Many more candidates left **part (b)** blank or gave incorrect answers such as quadrilateral or hexagon.

Answers: (a) Acute (b) Pentagon

#### **Question 8**

Many candidates found the function notation challenging but if they looked at a more familiar part, that of  $\frac{20}{x}$  and realised that the numbers in the brackets were factors of 20, that would have helped them realise that 4 and 5 were the missing numbers. Although many did not answer **part (a)**, out of those that attempted it, many got it correct. Some candidates noticed that when one of the numbers is divided into 20, the answer is always another of the numbers, as factors of 20 come in pairs. **Part (b)** was asking for the particular relationship for this function not the definition of domain and range. This relationship can be expressed simply as, 'they are the same numbers' and does not need complicated words or notation.

Answers: (a) 4, 5 (b) They are the same

# **Question 9**

Many candidates incorrectly answered 6 to **part (a)** as they did not take into account the shading of three of the vertices. For **part (b)**, often candidates drew six lines of symmetry or the correct three but with an extra vertical line.

Answers: (a) 3

# **Question 10**

This was one of the easier types of conversion questions. At the start of conversion questions candidates need to decide whether to multiple or divide by the exchange rate.

Answer. 540

#### **Question 11**

Candidates often confuse highest common factor with lowest common multiple and that was the case here with 12 being a frequent answer. One approach is to list the multiples of each number until the lowest common one is reached, which relies on accurate calculations. Another approach is to find each number as a product of prime factors and then to use these to formulate the product that will give the LCM.

Answer. 144



#### **Question 12**

In general, candidates could substitute into the equation but were not so successful at resolving the directed numbers.

Answer: 11

#### **Question 13**

Many candidates gave no workings and if their answer was even slightly incorrect, gained no marks. This question was best answered in two steps so credit could be given for one step correct. It is vital, as stated in the general comments, that workings are shown so candidates are able to gain marks for correct method.

Answer:  $\frac{py}{a}$ 

#### **Question 14**

Candidates needed to recall three pieces of information; alternate angles are equal, angles in a triangle equal 180° and two angles in an isosceles triangle are equal. Some candidates assumed *a* or angle *CAB* was equal to 40. A few candidates used a protractor even though the diagram was marked not to scale.

Answers: (a) 70 (b) 40

#### **Question 15**

Some candidates found this ratio and proportion question challenging. The common error was to think the sides of the larger triangle can be found by addition, so 17 (from 15-6+8) was the common incorrect answer. As with the previous question, some candidates measured rather than calculated. Some candidates did not show their working so it was difficult to know where many of the incorrect answers came from. The easiest way to proceed was to work out that the lengths in the larger triangle are 2.5 times the size of those in the smaller.

Answer, 20

## **Question 16**

This question was answered well by candidates, many of whom showed complete and convincing working. The first step was for candidates to convert to an improper fraction and the majority went on to show a correct method for division. Some candidates made arithmetical errors. Candidates were required to leave their answer as a fraction in its lowest terms and a decimal fraction was not acceptable.

Answer:  $\frac{18}{35}$ 

# **Question 17**

This was the question where candidates performed the best on the whole paper and there were very few answer lines left blank. The errors in **parts (a)** and **(b)** were mainly due to arithmetical slips such as 18 instead of 19 or –1 instead of –2. **Part (c)** was a different type of sequence and some did not recognise that the terms were the odd numbers squared so that the next term is 81 from 9<sup>2</sup>. Most who got this incorrect tried to do it by finding the sequence of the differences – in this case 8, 16 and 24 leading to the next difference being 32, then adding this 32 to 49 giving 81. This method is longer and has more places where arithmetical slips can be made. It is worth looking at a sequence as a whole rather than rushing in to find the difference between adjacent terms.

Answers: (a) 19 (b) -2 (c) 81

#### **Question 18**

As the scatter diagram had many plots, the negative correlation was very clear. Incorrect answers to **part (a)** included, down, indirect, positive and descriptions such as, price and distance from the city. Quite a few candidates left this blank. **Part (b)** required candidates to read from the diagram and many were able to give the correct answer of 4 km. Many drew an acceptable line of best fit in **part (c)(i)**. Candidates should use a ruler and pencil and not go over in pen afterwards. **Part (c)(ii)** was the best answered part of this question with a large majority giving an answer in the required range.

Answers: (a) Negative (b) 4 (c)(ii) 250 000 to 380 000

## **Question 19**

In general, this question was not answered very well and very often left blank. In **part (a)**, many understood what they were supposed to do as there was only one angle marked. There was a lack of accuracy in some drawings as some did not draw the proper construction with pairs of arcs but instead measured the angle with a protractor and drew the arcs in afterwards. Some just marked the angle. For **part (b)**, some used a ruler to find the centre of the line and then a protractor. However, some candidates did produce very neat well executed constructions.

## **Question 20**

This was one of the more complex questions on simultaneous equations as both equations needed to be multiplied to be in a position to eliminate one variable. There are various methods to solve simultaneous equations and candidates should be aware that sometimes, depending on the structure of the equations, one method might be quicker or involve fewer opportunities for arithmetic slips to spoil good method. Candidates should check their values in both equations.

Answer: (x =) -3, (y =) 7

#### **Question 21**

This area of the syllabus is often considered difficult by candidates. **Part (a)(i)** was a straightforward lead in to the work on the equation of a line and was answered well. Candidates struggled more with **part (a)(ii)**. They knew to use rise  $\div$  run but often ignored the scale of the grid giving, for example,  $6 \div 6$ . Candidates

who used the co-ordinates of two points on the line,  $\frac{y_2 - y_1}{x_2 - x_1}$ , were more likely to be correct. Many omitted

the equation of the line in **parts** (a)(iii) and (b) entirely. Some candidates in **part** (b) wrote y = 5x + b. Whilst this is written in the mark scheme, candidates must choose a value for c so answers such as y = 5x + 3, y = 5x or even y = 5x - 100 are all acceptable.

Answers: (a)(i) (0, 1) (ii) 2 (iii) (y = )2x + 1 (b) y = 5x + b

# **Question 22**

In general, candidates did well on the first two parts with fewer blank answers than some of the previous questions. **Part (a)** was a straightforward volume question, especially as there was a diagram. In **part (b)** candidates had to divide the volume by the given dimensions to find the third. This was slightly more challenging because of the lack of a diagram but **part (a)** should have led the candidates into the question. The very common error in **part (c)** was to treat this as a rectangular prism rather than a triangular one so a common incorrect answer was 1200, double the correct value.

Answers: (a) 672 (b) 12 (c) 600

# **MATHEMATICS (US)**

Paper 0444/21

Paper 2

## Key messages

To succeed in this paper candidates need to have completed full syllabus coverage, remember necessary formulae, show all necessary working clearly and use efficient methods of calculation.

## **General comments**

The level and variety of the paper was such that well prepared candidates were able to demonstrate their knowledge and ability. There was no evidence that candidates were short of time.

Candidates showed some good number work in **Questions 4** and **5** and were adept at dealing with negative numbers in **Question 7**.

Questions which caused significant problems were **Question 14** which involved recognising a cosine graph, converting units of speed to find a time in **Question 18**, the probability tree diagram in **Question 20**, the sector problem in **Question 22**, finding the equation of a perpendicular bisector of a line in **Question 25**, and trigonometry in the final question.

Candidates were generally good at showing workings; sometimes these were hard to follow, and should be set out in a logical manner, especially in an unstructured question. This was particularly prevalent in **Questions 18**, **22** and **25**.

Candidates should be aware that they would not be required to carry out complex calculations involving decimals or multiplying by  $\pi$  on a non-calculator paper.

# **Comments on specific questions**

## **Question 1**

Most candidates could deal with finding the time difference. Errors were commonly made in the number of hours, where 7, 9 and 12 were often seen. Many candidates treated the hours as if they contained 100 minutes and simply subtracted 732 from 2240 or 1040 leading to the commonly seen answers of 15 h 08 min and 3 h 08 min.

Answer: 8 (h) 52 (m)

#### Question 2

Most candidates found this percentages question straightforward. The most common incorrect method was to divide 25 by 3, leading to an answer of 8.3(3...). Some turned the question around and gave an answer of 88%. Many candidates showed a calculation of 3 divided by 25 rather than using the more efficient method of multiplying by 4.

Answer: 12

#### **Question 3**

The vast majority of candidates were able to convert the number from standard form. The most common error was to multiply rather than divide by 1000.

Answer: 0.00127

#### **Question 4**

The vast majority of candidates dealt with this question involving order of operations correctly. Where errors were made it was generally because they were simply working from left to right.

Answer: 28

# **Question 5**

Candidates were usually successful in this question and the vast majority gained at least 1 mark. Where there were arithmetic errors in the calculation, this usually involved the decimal point being incorrectly placed. There were some who divided rather than multiplied by the exchange rate which involved a time consuming calculation.

Answer: 540

#### **Question 6**

There was much confusion between factors and multiples in this question, with 2 and 12 commonly seen as answers. A common starting point was to list the factors of each number. A mark was often gained for making a correct first step of finding the prime factors of the numbers and sometimes for giving a multiple of the numbers which was not the lowest. A successful method employed was to draw up a table with both numbers at the top and then divide prime factors into each.

Answer: 144

## **Question 7**

The overwhelming majority of candidates dealt with substituting negative numbers correctly. If an error was made it was generally because -14 was arrived at from  $-2 \times -7$  leading to an answer of -17.

Answer: 11

# **Question 8**

This was a straightforward rearrangement which was dealt with completely correctly by the vast majority of candidates. There were various misconceptions, for example  $\frac{y}{x} = \frac{q}{p}$  followed by  $x = \frac{qy}{p}$ . Occasionally a subtraction was seen in place of a division.

Answer.  $\frac{py}{a}$ 

# **Question 9**

A sound knowledge of angles in triangles and parallel lines was demonstrated in this question with the majority gaining both marks. The most common misconception was to treat the isosceles triangle incorrectly and give both angles as 40°. Occasionally both angles were given as 70° and sometimes 50° or 140° were seen as subtractions of 40 from 90 or 180.

Answer: (a =) 70 (b =) 40

### **Question 10**

The majority of candidates found the correct solutions to the simultaneous equations. There were also many who had no strategy. Adding the two equations was the starting point for some who could then go no further. A number of candidates attempted to rearrange the equations to get *x* or *y* as the subject and wrote these rearrangements on the answer lines. Candidates would be advised to check that their values work in both equations and then they would realise if an error had occurred.

Answer: 
$$x = -2$$
,  $y = 7$ 

## **Question 11**

The majority of candidates knew the circle theorems involved in this question, with **part (a)** resulting in slightly more correct answers than **part (b)**. 56° was sometimes seen as the answer to **part (a)** demonstrating some confusion in the theorems. Other answers seen to **part (a)** were 236° (180 + 56) and 124° (180 – 56). The only common incorrect answer for **part (b)** was 28°. There was evidence of some measuring of angles from a minority of candidates, resulting in answers a few degrees out from the correct values. There was a fairly high proportion of answer lines left blank in this question, especially in **part (b)**, indicating that many candidates were unfamiliar with circle theorems.

## **Question 12**

There were a good number of candidates who could simplify the expression correctly and many gained 1 mark for a correct simplification of one part of it, commonly giving  $4p^4$  or  $16p^4$ . There were a range of other incorrect answers which stemmed from combining the indices and 16 in a variety of different ways.

Answer: 
$$2p^4$$

## **Question 13**

Solving the inequality was well attempted with many correct answers seen. There was also a large proportion who gained 1 mark either for collecting like terms on each side of the inequality or solving the equality and having the incorrect or no inequality sign in the answer. Those who rearranged to -4n < -15 often dealt with the negatives incorrectly, arriving at n < 3.75. Some spoilt their correct solution by choosing to write  $n = \frac{15}{4}$  or just  $\frac{15}{4}$  on the answer line.

Answer: 
$$n > \frac{15}{4}$$

# **Question 14**

This proved to be one of the most challenging questions on the paper with very few candidates giving a trigonometric function despite being told that it was one. There were an extremely high number of nil responses. Examples of the most common responses were f(x) = 2, 2x, 180, 360 and 180 + 2.

Answer: 
$$2\cos\frac{1}{2}x$$

## **Question 15**

Almost all candidates could give the next term in the sequence for **part** (a). Far fewer could give the *n*th term of the sequence required in **part** (b). The most common incorrect response was to give the term-to-term rule n-2. A common response which gained 1 mark was 7-2n. Candidates should be aware of the correct use of brackets with a decreasing terms sequence as some lost marks due to answers such as  $7-2 \times n-1$ .

Answer: (a) 
$$-3$$
 (b)  $9-2n$ 

#### **Question 16**

The majority of candidates demonstrated that they were adept at handling fractions and gained all 3 marks. Where full marks were not scored, 1 mark was usually gained by turning the mixed number into an improper fraction. The most common misconception was to invert the incorrect or both fractions.

Answer:  $\frac{18}{35}$ 

## **Question 17**

The majority of successful candidates employed the method of internal angles, finding the sum of angles of a hexagon and subtracting  $5 \times 115$ . Many candidates multiplied 5 by 115 correctly but then had no strategy for finding the sum of angles of a hexagon, with many thinking it was  $6 \times 180 = 1080$ . Incorrect answers generally involved working with  $115^{\circ}$  or multiples of this, and then adding or subtracting multiples of 180 or 360. Some candidates did try to work with the external angle and calculated 180 - 115 = 65 but then could get no further and gave 65 as their answer. 245 was also fairly common from 360 - 115. A significant number of candidates simply assumed that the polygon was regular and gave 115 as the answer.

Answer. 145

## **Question 18**

Only the most able candidates scored full marks on this question. It was also often difficult to follow working as it jumped around the answer space. Including the length of the car and the bridge caused some confusion with many not taking the length of the car into account, adding 2 lengths of the car, subtracting the length of the car, dividing or multiplying the length of the bridge by the length of the car. Other errors involved the use of an incorrect formula linking distance, speed and time, often leading to speed/distance, and errors in conversion between km and m or hours and seconds; the conversion was often done the wrong way or only one of the conversions was considered. Many candidates scored 1 mark for showing a clear distance divided by a speed. It would have been prudent for candidates to consider how realistic their answer was on this question.

Answer. 2

# **Question 19**

Some candidates were clearly proficient at questions on proportionality and were able to work through this question with relative ease. There were other candidates who made a good start and found that  $y = 4\sqrt{x}$ , but then forgot the square root when substituting  $\frac{1}{4}$  and obtained the answer 1. Some candidates were aware of proportionality, but worked with inverse proportionality or proportional to the square of x or proportional to x rather than the relationship given in the question. In other cases candidates did not know how to attempt this question and a wide range of incorrect answers were seen.

Answer. 2

# **Question 20**

It was evident that many candidates were not familiar with the use of tree diagrams and many appeared to have no strategy for answering the question. There were many cases of adding fractions along the branches, picking out fractions to add and multiplying 3 fractions together. Some candidates did not seem to worry if their answer was greater than 1. Of those who did use the tree diagram correctly, a common error was to omit the win, win option, giving the answer  $\frac{7}{12}$ . The most successful candidates used the most efficient method of 1 minus the lose, lose option.

Answer.  $\frac{5}{6}$ 

## **Question 21**

**Part (a)** was better attempted than **part (b)** where a reasonable proportion of candidates could simplify the surd. The most common incorrect attempt was to calculate a value and so 11.(...) was often the answer given. There were some good attempts at multiplying out the brackets in **part (b)** and some candidates gained 1 mark for a partially correct answer. The most problematic part of the multiplication was  $2\sqrt{5} \times 3\sqrt{5}$  which most commonly resulted in  $6\sqrt{5}$ , alongside 6,  $6 \times 25$ ,  $6 \times 5\sqrt{5}$  and  $6\sqrt{10}$ . Another common misconception was to combine each set of brackets to  $(5\sqrt{5})(-1\sqrt{5})$ . Another fairly common error was to only multiply the corresponding terms in each bracket, leading to an answer of  $6 - 6\sqrt{25}$ .

Answer: (a)  $5\sqrt{5}$  (b)  $-24-5\sqrt{5}$ 

#### **Question 22**

The crucial step to solving this problem was to find the sector as a fraction of the circle using the length of the arc and radius. Those who made this connection usually went on to find the correct answer. Some lost the final accuracy mark through multiplying out  $\pi$  throughout the question unnecessarily. Many candidates did not show anything which was worthy of any credit, including statements and calculations about the circumference and area of the whole circle. This often involved multiplying and dividing by  $\pi$  and ending up where they began, with answers of 81, 18 and 54 for example. There were a significant percentage of candidates who did not attempt this question.

Answer: 27

#### **Question 23**

This was well attempted with many candidates scoring full marks and those who did not often scored 2 marks for identifying that the vertical height was 3. Some candidates used Pythagoras' theorem incorrectly and added the values rather than subtracting, but many who did get an incorrect height gained a mark for using this correctly to find the area of their trapezium. Some candidates assumed that the height was 4 but as long as this was clearly marked on the diagram, could gain the mark for correctly finding the area of their trapezium. Others incorrectly used the slant height of 5 or simply multiplied all 3 given values together.

Answer, 30

#### **Question 24**

In **part (a)** a large number of candidates were able to fully factorise the expression, and there were also a good number who gained 1 mark for a partial factorisation, but then did not realise that they could combine the terms outside each bracket into another bracket, or made an incorrect attempt. Some believed that the expression could not be factorised or made an incorrect attempt at collecting together the terms. In **part (b)** it was rare to see full marks. A reasonable number of candidates were able to gain 1 mark for a partial factorisation, usually for  $2(81-4t^2)$ , but did not recognise that it could be factorised further as the difference of two squares. There was a fairly high number of nil responses for **part (b)**.

Answer. (a) (a + 2)(2 + p) (b) 2(9 + 2t)(9 - 2t)

## **Question 25**

It was rare to see a fully correct response to this question, but there were many candidates who were able to gain some marks. A good number were able to find the gradient of the line but far fewer went on to find the negative reciprocal of their gradient with some inverting but not making it negative and others vice versa. Where candidates found a gradient which they believed was for a perpendicular line they often did not find and use the midpoint in attempting to determine the constant term and instead substituted one of the two points given. Many substituted a point into a linear equation using the gradient of the original line rather than the perpendicular, even though they often gave this inverted gradient within the final answer. A smaller number of candidates found the midpoint of the line, but then did not progress from this point. An error in finding the midpoint was to subtract the co-ordinates and divide by 2, resulting in (3, 7). Other common errors included attempts to draw a graph (despite the lack of graph paper), calculations for gradient being inverted and a range of incorrect calculations using the values in the co-ordinate pairs given, including finding the length of the line. A relatively high proportion of candidates did not attempt the question at all.

Answer: 
$$y = -\frac{3}{7}x + 11$$

#### **Question 26**

Both parts of this question were impacted by candidates incorrectly making one of two assumptions, the first being that the triangle was right-angled. In **part (a)** this led to candidates attempting to use  $\frac{1}{2}$  base x height with their value of AC.  $\frac{1}{2} \times 8 \times 3$  was also commonly seen.  $\frac{1}{2} \times 7 \times 10$ . In **part (b)** this assumption led to attempts using Pythagoras' theorem. The second incorrect assumption made was that the triangle was isosceles and so the perpendicular height cut BC at 4 cm. Pythagoras' theorem was then used to find the height and subsequently the area in **part (a)** and to find AC in **part (b)**. There were, however, a good number of candidates who recognised the need to use the efficient formula involving sine for **part (a)** and the cosine rule in **part (b)**. There were very few though who could recall the values of sine or cosine of 60. There were a high number of nil responses in this question, particularly in **part (b)**.

Answer: (a)  $6\sqrt{3}$  (b) 7

# **MATHEMATICS (US)**

Paper 0444/31 Paper 3

## **Key messages**

To succeed in this paper candidates need to have completed full syllabus coverage, remember necessary formula, show all working clearly and use a suitable level of accuracy.

## **General comments**

This paper gave all candidates an opportunity to demonstrate their knowledge and application of mathematics. Most candidates were able to complete the paper in the allotted time. Few candidates omitted part or whole questions. The standard of presentation was generally good and there was evidence that most candidates were using the correct equipment.

Candidates continue to improve in showing their workings and gaining method marks. Centres should encourage candidates to show their working clearly, explicitly showing, for example, which values they are multiplying or dividing together.

Many candidates were unable to gain marks in the 'show that' question if they used the value they had to show from the beginning.

Attention should be paid to the degree of accuracy required in each question and candidates should be encouraged to avoid premature rounding in workings. Candidates should also be encouraged to process calculations fully and to read questions again once they have reached a solution so that they provide the answer in the format being asked for and answer the question set.

## Comments on specific questions

#### **Question 1**

Many aspects of number work were examined in this question. It gave all candidates an opportunity to show their understanding of number, including factors, multiples, percentages and fractions. All candidates were able to attempt all or parts of this question.

- (a) (i) The majority of candidates were able to choose the correct number from the list. A large proportion of candidates however confused multiples for factors and gave the most common incorrect answer of 30. Some candidates did not read the question carefully enough and gave a factor of 15 which was not in the list, often 5.
  - (ii) The majority of candidates gave the correct multiple from the list. Sometimes those that had confused factor for multiple in **part** (i) gave the answer of 6, a factor of 18.
  - (iii) Candidates demonstrated their understanding of square numbers well, with the majority of candidates giving the correct answer. However the importance of rereading the question is emphasised here as some candidates gave 36 as their answer.
  - (iv) Candidates were more successful in finding the cube number from the list with the majority giving the correct value. The most common incorrect answers were 3 and 49.

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- (v) Many candidates were able to correctly identify the cube root of 216 as 6. However a significant proportion of candidates chose not to attempt this question, likely because they did not know how to use their calculators to find a cube root.
- (b) (i) Candidates were very successful in this part with the vast majority gaining full marks.
  - (ii) Candidates again found this question straightforward with the vast majority of candidates giving the correct answer. Some less able candidates gave their answer as a decimal.
- (c) All candidates were able to attempt this question with the majority correctly simplifying the fraction to its lowest terms. All candidates showed some understanding of simplifying although many less able candidates did not gain full marks as they did not simplify to the lowest terms, often leaving their answer as  $\frac{14}{21}$ .
- (d) (i) A wide range of methods were used successfully to write 45 as a product of its prime factors. The most common and successful was using a table or tree to find the prime factors and then give the answer as a product of these factors. Often correct tables and trees were seen but then answers were not given as a product; the most common was lists, e.g. 3, 3, 5. Some candidates scored one mark for a correct product that equalled 45, e.g.  $3 \times 15$  or  $5 \times 9$ .
  - (ii) Few candidates found the LCM, however 315 and 4725 (45 × 105) were seen as answers from some candidates. The most common successful solutions wrote 105 as a product of prime factors and then used their answer to **part (i)** to find the HCF. However a large number of candidates who had not gained marks in **part (i)** still gained full marks in **part (ii)** by listing factors of 45 and 105. Although these lists were often not complete they were able to identify 15 as the HCF or gained one mark for identifying 5 or 3 as a common factor.

Answers: (a)(i) 3 (ii) 36 or 72 (iii) 49 (iv) 27 (v) 6 (b)(i) 43 (ii) 50 (c)  $\frac{2}{3}$  (d)(i)  $3 \times 3 \times 5$  (ii) 15

## **Question 2**

To be successful in this question, candidates had to demonstrate a good understanding of probability, averages and pie charts. This question also contained a 'show that' question which candidates found very challenging.

- (a) (i) Very few candidates gave probabilities as ratios (which are not acceptable) and those that chose to express as a percentage or decimal generally were successful. The majority of candidates were able to count the correct number of odd numbers and give their answer as the correct fraction. The most common incorrect answer was  $\frac{3}{5}$ , from counting even numbers rather than odd numbers.
  - (ii) Candidates found this question more challenging and often gave the answer of  $\frac{2}{5}$ , most probably from not recognising 2 as a prime number. A large number of candidates reversed the answers to parts (i) and (ii).
  - (iii) This part was the most successfully answered of this question. The vast majority of candidates recognised that the spinner had no number 7 and gave their probability in the correct form.
- (b) (i) Candidates were required to identify the mode from a list of values. Most candidates immediately identified 4 as the correct answer as it has the highest frequency. Less able candidates however showed little understanding of mode and calculated the median or gave 6 as their answer as this was the largest number in the list, rather than most frequent.
  - (ii) Candidates continue to improve from previous years in finding the mean from a list of values. Very few candidates calculated the median instead of the mean. The most common error was making numeric errors in adding all 20 values accurately.

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- (iii)(a) Candidates were required to show that the sector of a pie chart constructed from the results in the table would have a sector angle of 54° for the number 2. This part was only successfully answered by the most able candidates due to the requirements of a 'show that' question. It is important to emphasise to candidates that in questions of this type, candidates must not use the figure they are trying to show, i.e. 54°. A larger proportion of candidates chose not to attempt this question or to draw the sector angle of 54°. Successful answers were attempted in two parts. Most candidates first calculated that one spin was equivalent to 18° (360 ÷ 20) and then multiplied by 3. Clear and explicit working out is required in these questions.
  - (b) Candidates who had answered the previous part successfully were generally also able to gain full marks in this part. Candidates who had not attempted part (iii)(a) often did not attempt this part also. Some candidates seemed to answer this part after continuing with the question and used the value of 168° which came from the second pie chart given in part (c). Candidates should be aware that all information required to answer a question will be provided in the part they are answering or in previous parts, not in subsequent questions.
- (c) (i) A number of correct methods were seen, the most common finding the fraction or percentage of the pie chart and then multiplying by the total number of students. The most common incorrect answer was 28, found by dividing 168 by 6.
  - (ii) This part of the question also proved to be challenging for candidates. Many did not use the information given in bold in the question and often attempted the percentage of candidates guessing a number less than 6. Successful solutions showed the total angle for sections 2, 3 and 4 and a correct conversion from an angle to a percentage. This was often done in individual parts for 2, 3 and 4 and then added together.
  - (iii) Candidates were more successful in identifying that the sector for number 5 represented 10% of the students. However, more commonly, candidates showed that 10% of the students was 3 but often then concluded that this was the sector for the number 3 rather than continuing to work out the angle for 3 students. Similarly, candidates often found 10% of the circle as 36° (gaining a method mark) but then did not measure the pie chart to find which sector this was. Often, less able candidates simply guessed at a sector, showing no working, and the majority choosing the sector for the number 2 as this was the smallest.

Answers: (a)(i) 
$$\frac{2}{5}$$
 (ii)  $\frac{3}{5}$  (iii) 0 (b)(i) 4 (ii) 4.3 (iii)(a)  $\frac{3}{20} \times 360$  (iii)(b) 90 (c)(i) 14 (ii) 43.3 (iii) 5

## **Question 3**

Understanding of speed, time, money and ratio were essential skills tested in this question.

- Good solutions to this question were given in stages. The most common method was to calculate the cost of the 14 nights for 2 people  $(237 \times 14 \times 2)$ , then find 6% of this total and finally add it. This method could have been done in any order and all possible combinations were seen. Often candidates found calculating the 6% difficult with many incorrect methods seen. However these candidates often gained one mark for completing the other multiplications correctly. The importance of showing working is again highlighted here, as many candidates simply gave a final answer which would have gained a part mark if they had shown the multiplications used.
- (b) Calculating the change was the most successfully answered part of this question. Often this was seen with no working. A common incorrect answer was \$12.11, from only subtracting one bottle from \$20. Again candidates should be encouraged to reread questions after they have given their answer to check they have read all given information correctly.

- (c) The key to answering this question successfully was the knowledge that 1 hour = 60 minutes and therefore  $\frac{3}{4}$  hour = 45 minutes. Many less able candidates used 75 minutes and therefore gave the incorrect answer of 16 38. Equally important to gain full marks was the ability to work in 24-hour times. Many candidates gave their answer as 4 08 which only gained one mark as the required answer had to be in 24-hour time or, if given in 12-hour time, had the appropriate pm. Many candidates used a column method of adding times and then converting times over 60 minutes to hours and minutes. This proved a successful method for some candidates but many errors were seen when converting from minutes to hours and minutes to gain the final answer.
- (d) Good solutions gave clear workings out, showing a calculation of time from the correct formula, converting this time in hours to hours and minutes, and then finally adding this time correctly and giving the solution in 24-hour time or 12-hour time with am included. Only the most able candidates could complete all parts successfully with the majority of candidates making one or more errors. The most common error was converting their time to hours and minutes. The majority of candidates calculated the time to be 8.3 hours or 8.33 hours. However, depending on the degree of accuracy, these values often became 8 hours 30 mins or 8 hours 33 mins. The accuracy was vital in gaining full marks as often candidates used a correct method in converting hours to hours and minutes but because they had only used 2 significant figures, their answer of 8.3 hours became 8 hours 18 minutes and subsequently a final solution of 02 58 was seen often.
- (e) The majority of candidates showed good understanding of ratio and were able to find the correct number of male passengers. This was commonly found by dividing by 9 and then multiplying by 5. The most common incorrect methods used were dividing 1800 by 5 (giving 360 as the final answer) or dividing by 5 and then multiplying by 4 (giving 1440 as the final answer).

Answers: (a) 7034.16 (b) 4.22 (c) 1608 (d) 0300 (e) 1000

#### **Question 4**

This algebra question was the most successfully answered of the whole paper. Candidates showed very good ability to form and solve equations.

- (a) (i) Solving this one-step equation proved to be one of the most successful questions of the whole paper. Very few candidates needed to show working out and nearly all candidates found the correct answer.
  - (ii) This more complex equation was more difficult to solve but the vast majority of candidates solved it correctly. Many gave the correct answer with no working. However the majority showed good algebraic skills, successfully expanding the bracket, subtracting 40 from both sides and then dividing by 15. Few candidates attempted the other possible method, divide by 5, subtract 8 and finally divide by 3. Candidates who attempted this method were generally more able candidates who did it correctly.
- (b) (i) Candidates were able to form the correct function from the information given in the question. Good reading skills were shown and the example given in the question helped the less able candidates form the correct expression. Very few incorrect answers were seen and even fewer candidates chose not to attempt this question.
  - (ii) This part was the most challenging part of this algebra question. The best solutions equated the two expressions in **part** (i) and gave a thorough algebraic solution. The most common error was to start incorrectly by not equating the expressions but to form two separate equations and attempt to solve simultaneously. This often led to errors, by adding the *x* terms and forming an incorrect equation of 34x = 742. Candidates who correctly solved this equation were able to gain one mark for a correct solution of a wrong equation.

Answers: (a)(i) 8 (ii) -2 (b)(i) 19x + 117 (ii) 127

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## **Question 5**

All candidates were able to attempt all or part of this question which assessed candidates' ability to work with negative numbers, rounding, estimation and area of circles in context.

- (a) (i) This part was one of the most successfully answered in the whole paper. The most common incorrect answer was  $-3^{\circ}$ C, the temperature instead of the day.
  - (ii) This part again proved to be successful, with the vast majority of candidates finding the difference as 5 °C or –5 °C (both answers were acceptable).
  - (iii) This part was also one of the most successfully answered in the whole paper. Nearly all candidates successfully wrote the temperatures in the correct order. The only common error seen was candidates starting with the highest temperature instead of lowest as instructed in the question.
  - (iv) The majority of candidates correctly subtracted 4 °C from –2 °C. Common incorrect answers were 10 °C (from adding 6 °C and 4 °C) or 2 °C (from adding 4 °C to –2 °C).
- **(b) (i)** The correct answer was given by most candidates in acceptable forms, two million or 2 000 000. The most common error was rounding to a higher degree of accuracy, often to the nearest 1000 or 100.
  - (ii) Good solutions to this question demonstrated candidates' understanding of what an appropriate level of accuracy means. Successful candidates rounded the figure given in **part** (ii) to the nearest million and then divided by their answer in **part** (i). Many less able candidates chose not to attempt this question.
- Candidates were challenged in this question to calculate areas of circles in the context of a cross section of a circular tunnel. The best solutions were completed in stages clearly showing workings out throughout. The most successful candidates quoted the formula for the area of a circle and gave the radii of both circles before calculating the respective area. Very few candidates lost marks for using 3.14 or  $\frac{22}{7}$  for their value of  $\pi$ , with the vast majority of candidates using the  $\pi$  button on their calculators. Some candidates however lost a mark for rounding prematurely, before subtracting, and therefore found an answer outside the accepted range. Many candidates were able to gain some of the marks for calculating the area of the inside circle using the radius of 4 m. However many candidates then calculated the radius of the larger circle to be 4.5 cm (from 8 m + 1 m) and found an incorrect second area. Very few candidates did not know or use the correct formula for the area of a circle.

Answers: (a)(i) Wednesday (ii) 5 (iii) -3, -2, -1, 0, 1, 2, 5 (iv) -6 (b)(i) 2 million (ii) 3 (c) 28.3

# **Question 6**

This angles and scale drawing question offered candidates the opportunity to show they could measure and draw bearings on a scale drawing, calculate a missing angle and use Pythagoras' theorem to calculate a missing length in a right-angled triangle.

- (a) (i) Candidates continue to find measuring bearings very challenging. The majority of candidates showed little understanding of bearings with the most common answer being a measurement of length (9 cm) rather than an angle. Very few candidates showed the ability to use a protractor accurately when measuring a bearing.
  - (ii) Candidates were far more successful at measuring the length of port *A* to port *B* and using the scale to find the actual distance. A few candidates did not convert from 9 cm to 135 km, although they still gained one mark for a correctly measured length in cm.
  - (iii) Drawing bearings proved equally challenging for nearly all candidates. However in this part candidates generally gained one mark for correctly drawing port *C* 6 cm away from port *B*. This was however drawn in a variety of directions from port *B* and very rarely on a bearing of 146°.
- **(b) (i)** Candidates demonstrated their knowledge of angles in a triangle adding to 180°. The vast majority of candidates successfully found the correct answer.

- (ii) Calculating a bearing from a diagram proved to be the most challenging question on the whole paper, with only a few correct answers seen. A variety of incorrect answers were seen, using a variety of angles given on the diagram. Many candidates used correctly the 43° and 29° but subtracted these from 360° instead of adding to 180°. The answer of 255° was seen from measuring the diagram. Candidates should be reminded that this diagram was clearly labelled NOT TO SCALE and a calculation is required rather than a measurement.
- (c) Candidates have shown an improvement in identifying the use of Pythagoras' theorem from previous years. The majority of candidates correctly squared, added and then square rooted. Good solutions showed all steps of the working. Very few candidates subtracted instead of added. Candidates who did not use Pythagoras' theorem generally added or subtracted the lengths hence giving the incorrect answers of 623 km or 89 km.

Answers: (a)(i) [0]67 (ii) 135 (b)(i) 29 (ii) 252 (c) 445

#### **Question 7**

This question challenged many candidates as it assessed some more complex parts of the syllabus, including trigonometry, compound interest and percentage change. Candidates who showed thorough working were more successful in all parts.

- (a) (i) The best solutions seen gave clear and thorough working out. Candidates who made markings on the diagram were generally more successful in finding the three separate areas and then adding. Candidates should be encouraged to write on the diagram to mark in missing lengths and to draw lines to split the compound shape into separate rectangles. A variety of methods were successfully used, all with clear workings out. A few candidates gave the correct answer with no working but this was very rare; most answers with no working were incorrect. Candidates who did not find one of the two missing required lengths gained no marks. The most common error was to calculate two correct rectangles (usually  $7.5 \times 3.2 \times 2$ ) but then to make errors on the third (usually  $11.8 \times 4.7$  instead of  $5.4 \times 4.7$ ).
  - (ii) Candidates were able to gain full marks even if they had not calculated the area correctly in part (i). Good answers showed their area from part (i) multiplied by 2175 and then this figure correctly rounded to 3 significant figures. However often only the answer was given, with no multiplication seen, and if the candidate had incorrectly rounded then no marks were given. This question emphasises the need to show all stages of working out as an incorrect rounding could still gain one mark if the correct multiplication had been seen. Most candidates were able to gain one mark for a correct multiplication. However rounding to 3 significant figures was only completed correctly by more able candidates.
- (b) Candidates have shown an improvement, compared to previous years, in the use of trigonometry. A greater number can identify the correct trigonometric ratio to use and then substitute correctly into it. More able candidates can then generally go further and find the angle by using the inverse tangent button on their calculators. A large proportion of candidates could identify and substitute into the tangent ratio but left their answer as 1.02... (1.8 ÷ 1.75). Few candidates used the incorrect trigonometric ratio or substituted the lengths incorrectly (e.g. 1.75 ÷ 1.8). Less able candidates generally chose not to attempt this question.
- (c) Good solutions to this question quoted and substituted values into the formula for compound interest or each year was calculated separately. Some correct answers were spoilt by rounding to the nearest dollar but this was rare. The most common error was to calculate simple interest, common incorrect answers being 53 000 or 3000.
- (d) An improvement was seen in calculating percentage profit compared to previous years. This could be because the percentage profit was 10% which many able candidates could spot without showing any working out. Most candidates were able to gain one mark for showing the profit was \$18 000, however the most common error was then to divide by \$198 000 instead of \$180 000.

Answers: (a)(i) 73.38 (ii) 160000 (b) 45.8 (c) 53060.4[0] (d) 10

## **Question 8**

This question gave candidates the opportunity to demonstrate their ability to calculate missing values and draw a quadratic curve.

- (a) Most candidates correctly calculated the missing values in the table. Very few candidates did not attempt this part.
- (b) Candidates showed good skills of plotting points correctly from their table in **part (a)**. Most candidates gained three of the four marks available for correctly plotting all nine points. The most common error which lost candidates the final mark was to join the points with line segments or to draw a smooth curve but join the top two points with a straight line.
- Giving the co-ordinates of the highest point on the curve proved to be one of the most challenging questions of the whole paper. Candidates needed to recognise the symmetry of their graph to realise that the *x* co-ordinate had to be 3.5. Many candidates quoted the highest value from their table (3, 20) or (4, 20) or had drawn a straight line at the top of their curve which resulted in the same incorrect answers.
- (d) (i) Many correct ruled lines were seen. However a large proportion of candidates did not gain the mark as the line was not drawn with a ruler or did not go across the whole grid. Candidates should be reminded that straight line graphs should always be drawn with a ruler.
  - (ii) Good solutions to this question followed an accurate drawing of the line y = 16. Some candidates misread the scale and used 1 square = 0.1 instead of 1 square = 0.2. A large number of candidates attempted to use the quadratic formula or factorisation despite the question requiring the line to be used. Very few correct answers were found using the formula or factorisation due to the need of rearrangement first.

Answers: (a) 14, 20, 20, 14, 0 (c) (3.5, h) where  $20 < h \le 20.4$  (d)(ii) 1.4 5.6

# **Question 9**

This question assessed candidates' ability to perform an enlargement and translation of an image on a grid and to describe fully a reflection and rotation.

- (a) This part was the best attempted by candidates. Most candidates gained at least one mark for correctly translating the original shape 2 to the left or 6 down. The best solutions did both with a clear image drawn with a ruler. Some less able candidates rotated the shape or reflected it.
- (b) (i) Fewer candidates attempted this question with many incorrect answers seen. Most candidates enlarged from the correct point and drew at least 2 points correctly but few candidates gave the fully correct answer. The correct image was seen but in the incorrect position by a small number of candidates who gained one mark for an enlargement of scale factor 2.
  - (ii) This part proved to be one of the most challenging of the whole paper with a large proportion of candidates choosing not to attempt it. The most common answer seen was –2.
- Candidates found describing a reflection easier than a rotation in **part (d)**. Most candidates identified the transformation as a reflection but few candidates could then go on to correctly describe the mirror line as x = -1. The equation of the mirror line was often given as the *y*-axis or y = -1.
- (d) Good answers contained all three parts to describe a rotation, including angle and centre of rotation. The most common error was to omit the centre of rotation. Less able candidates could correctly identify the transformation as rotation but did not include the direction or centre.

Answers: **(b)(ii)**  $\frac{1}{2}$  **(c)** Reflection, x = -1 **(d)** Rotation, [centre] (0,0), [angle] 180°

# **MATHEMATICS (US)**

Paper 0444/41 Paper 4

# Key messages

To do well in this paper candidates need to be familiar with and practiced in all aspects of the syllabus. The accurate statement and application of formulae in varying situations is always required. Work should be clearly and concisely expressed with an appropriate level of accuracy.

## **General comments**

Candidates appeared to have sufficient time to complete the paper and any omissions were due to lack of familiarity with the topic or difficulty with the question rather than lack of time. More able candidates could attempt all the questions and solutions usually displayed clear methods. However, some candidates provided solutions with little or no working or didn't carry out calculations to sufficient accuracy and consequently lost marks. Centres should continue to encourage candidates to show all working clearly in the answer space provided. Some candidates risk losing marks by overwriting incorrect working which then becomes unclear. For questions requiring several calculations candidates are advised to write down the answer to each step using more than 3 significant figures and only correct to the required accuracy at the end of the calculation.

The topics that proved to be more accessible were percentages, ratio, and inverse functions. The more challenging topics were probability based on group data, questions requiring candidates to show a solution, vectors and using and interpreting a drawn graph.

# **Comments on specific questions**

# **Question 1**

- (a) Most candidates were able to draw the correct image with a few earning one mark for a translation with either the correct horizontal displacement or correct vertical displacement.
- (b) Most candidates earned full marks for a correct reflection. Common errors involved reflection in other horizontal lines, often the x-axis, or in the line x = 1.
- (c) Candidates were slightly less successful describing the enlargement, largely due to an incorrect centre. There were a significant number of candidates who ignored 'single' and gave two transformations.
- (d) There was a lot of confusion between the transformations 'stretch' and 'enlargement' and also the use of 'x-axis invariant' and 'y-axis invariant'. Some omitted the word invariant.

Answers: (c) Enlargement, scale factor = 3, centre (-6, -5) (d) Stretch, scale factor 2, x-axis invariant

# **Question 2**

- (a) (i) This question was almost always correct. Errors usually involved division by 2 or by 5 and occasionally forgetting to multiply by 2 after division by 3.
  - (ii) This was another part that was almost always correct. The usual error was to subtract 32.40 from 72 to give the amount left.
  - (iii) Far fewer candidates reached a correct final answer in this part, often struggling to cancel  $\frac{31.2}{72}$  correctly. Common errors usually involved subtraction of 8.40 from 32.40, instead of adding, before finding the result as a fraction of 72.
  - (iv) Many correct answers were seen. The most frequent error was to increase 19.2 by 20% leading to an answer of 23.04.
- (b) There were many correct answers to this question, with the usual formula quoted to find the \$110 interest. However some candidates forgot to add this to \$550 for the final amount and lost marks. Some were clearly confused and attempted a compound interest method.
- (c) Many candidates showed some working and often gained credit for doing so. Those candidates who did year-on-year calculations and wrote down the total at each stage rarely reached a correct answer, largely due to rounding and/or truncation errors.
- (d) Although many candidates were able to set up a starting equation of the form  $550 \times m^{10} = 638.30$ , solving it proved challenging for all but the more able candidates. Understanding of the order of operations proved the downfall of many, often starting by subtracting 550. Those with a multiplier of  $(1 + r)^{10}$  experienced the same difficulties, as well as subtracting 1 from both sides. Attempts at a trial and improvement method rarely ended with a correct answer.

Answers: (a)(i) 48 (ii) 32.40 (iii)  $\frac{13}{30}$  (iv) 24 (b) 660 (c) 663.90 (d) 1.5

## **Question 3**

- (a) (i) Many correct answers were seen but a number of candidates gave an answer of 300, presumably as it is halfway between the minimum and maximum volumes.
  - (ii) This part was well answered by the majority with incorrect answers such as 100, 125, 150 and 200 seen.
  - (iii) This proved to be the most challenging of the four parts as evidenced by the higher number making no attempt. Able candidates had no problems but less able candidates had little idea of where to start reading from the graph with no obvious pattern to the many incorrect answers.
  - (iv) Candidates were more successful in this part with most candidates earning full marks. There were two main reasons for the loss of marks; misreading the vertical scale and giving the number of students estimating less than 300 m<sup>3</sup>.

- (b) (i) This was generally well answered, with marks lost occasionally due to poor arithmetic or not using the correct mid-values. Some had a partial understanding and multiplying the frequencies by the class widths or the bounds of the interval were reasonably common errors. A small number simply added the mid-values and divided by 4.
  - (ii) A small majority of candidates were able to draw a correct histogram. Others lost marks for incorrect widths of bars, usually the first which was often drawn over the interval 0 to 60. When bars were drawn incorrectly it was very rare to award a mark for 3 correct frequency densities; in most cases no working was shown.
  - (iii) Many candidates struggled to obtain the correct answer. Many solutions were based on choosing only one student or choosing two with replacement.

Answers: (a)(i) 400 (ii) 350 (iii) 70 (iv) 170 (b)(i) 106 (iii)  $\frac{1339}{4975}$ 

#### **Question 4**

- (a) Many correct answers were seen. Some candidates didn't use the required values for  $\pi$  in this part and in others, which led to inaccurate answers and the loss of some marks. In a few cases, candidates copied the formula incorrectly. Some clear thinking candidates simplified their working throughout the question by using multiples of  $\pi$ , only converting to a decimal for the answer to a part. Some candidates showed a correct calculation and wrote the answer as 14 140. To earn both marks, the answer that rounded to 14 140 needed to be written down.
- (b) (i) Candidates were less successful with this part. A majority of candidates realised that the sphere was taking up space where water would have been and proceeded to calculate the volume of the cylinder and subtract their previous answer. Errors arose when a variety of incorrect formulae were used for the cylinder. Some used their incorrect answer from **part** (a) rather than using the volume given in the question.
  - (ii) Many candidates didn't link the drop in water level to the removal of the sphere resulting in a greater number of candidates making no attempt at this part. For the rest, the most common approach was to equate the volume of water from **part** (i) to the volume of a cylinder of water of height *d*. A smaller number attempted the drop in height when the sphere was removed although not all went on to subtract the answer from 60.
- (c) (i) A majority of candidates attempted to equate the volume of the cone to the volume of the sphere from **part** (a). Although a significant number were successful, much of the working was littered with errors. Misreading the formula with  $\frac{1}{3}$  becoming either  $\frac{4}{3}$  or  $\frac{1}{2}$  and  $r^2$  becoming  $r^3$  resulted in lost marks for many. Accurate equations were often not solved correctly, dealing with the fraction or the square, and the order of operations being the common errors.
  - (ii) Using perpendicular height in place of slant height and not including the circular base were common errors and resulted in only a minority of candidates obtaining the correct answer. There was also a lack of working shown in this part, even when the answer was correct. Throughout the question candidates needed to use answers from previous parts and this often led to the loss of accuracy marks.

Answers: (a) 14 137 (b)(i) 104 000 (ii) 52.8 (c)(i) 15.8 (ii) 3580

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## **Question 5**

- (a) The two missing values were almost always correct.
- (b) The scales used on the axes made the plotting of the points more challenging and it was common to see the points (1.6, 14.1) and (8, 10.5) and their negative counterparts plotted incorrectly. Also, the tight turns on the two sections of the graph led to less than perfect curves. Candidates would be well advised to use a sharp pencil for plotting points and the drawing of the curve. Many were seen with thick lines. Some candidates plotted two symmetrical curves below the *x*-axis. However most candidates realised that the value of f(x) tended towards infinity at x = 0 and, consequently, most curves did not cross or touch the *y*-axis.
- (c) It was clear that many candidates understood what was required, but correct results depended on the quality and accuracy of the curve in **part** (b). Consequently, some marks were lost as solutions often fell outside the required range.
- (d) This proved challenging for many and a large number of candidates made no attempt. Some fully correct answers were seen but a combination of missing values, extra values and numbers that were not prime, meant that the modal mark was zero. Common extra numbers often included 0 or 1 or both and extra primes usually involved 13 and 17. Some candidates included negative values for *k*.
- (e) A small majority picked up on the symmetry of the two sections of curve and gave the correct point. Some were unsure and answers such as (2, -12) and (-2, 12) were seen along with more random co-ordinates.
- (f) (i) All four parts proved challenging for many and a significant number of candidates made no attempt. In this part, some realised the need to equate  $\frac{20}{x} + x$  with  $x^2$  and were able to show the given result. Elimination of the denominator often resulted in the very common error of  $20 + x = x^3$ . Many tried to solve or rearrange the cubic equation using the values -1 and -20.
  - (ii) Very few fully correct parabolas were seen. Most realised that it was a U shape, but some did not place it through (0, 0). Simple plots such as (1, 1), (-3, 9) and (3, 9) were often missed. A few candidates plotted points at (-1, -1), (-2, -4), etc.
  - (iii) The accuracy here depended on good curves for the two graphs and, where this was achieved, candidates produced some good answers, clearly understanding that the intersection of these was required.
  - (iv) Nearly all candidates didn't relate this to **part** (iii). Some attempts at trial and improvement were seen but were rarely successful. The most common incorrect answer was 20.

Answers: (a) 9, 10.5 (c) 2.1 to 2.6, 8.5 to 9 (d) 2, 3, 5, 7 (e) (-2, -12) (f)(iii) 2.5 to 3.5 (iv) 3.0 to 3.1

# **Question 6**

- (a) (i) As with many 'show that' questions, many candidates didn't realise the steps required and, as a result, many did not gain marks. Able candidates had no difficulty in obtaining an expression, in terms of x, for the length of the rectangle, either by considering the perimeter or the area. Once this was found it was usually a couple of steps to obtain the correct equation. Less able candidates often took the given equation and tried, unsuccessfully, to work backwards. Some candidates, wrongly, substituted their own numerical values and tried to 'solve' their resulting equations.
  - (ii) A majority of candidates were able to obtain the correct solutions, many by using the quadratic formula resulting in a loss of marks. Whether this was a consequence of not reading the question carefully or an inability to factorise was not clear.
  - (iii) A majority of candidates made a good attempt and usually obtained two correct solutions, though not always written to two decimal places as requested. When using this method some candidates made sign errors, using –40 instead of –(–40) for –b. Others squared –40 to obtain –1600 and some shortened the division line so that only the square root expression was divided by 2.

- (b) (i) Most candidates correctly gave the times taken as  $\frac{200}{x}$  and/or  $\frac{200}{x+10}$ . Some subtracted correctly and were able to do the algebraic manipulation to obtain the required result. Many, however, lost marks for an incorrect reverse subtraction.
  - (ii) Two methods were used, substituting 80 into the expression from the previous part or starting fresh and working out the times for the individual journeys. Those opting for the first method made errors such as 80(80+10) = 6410 or in converting their time in hours into minutes and seconds. The conversion also caused many errors for those attempting to find the time for the reverse journey.

Answers: (a)(ii) (x - 30)(x - 10) and 30, 10 (iii) 5.86, 34.14 (b)(ii) 16 min 40 s

## **Question 7**

- (a) (i) Many correct answers were seen, usually recognising that opposite sides are represented by the same vector. Some preferred to use a route such as  $\overline{MR} + \overline{RO} + \overline{OP}$  to find  $\overline{MQ}$ , although not all went on to write it in terms of **p** and **r**.
  - (ii) Candidates were less successful in this part. Most chose to use the route  $\overline{MQ} + \overline{QT}$ , but obtaining an expression for  $\overline{QT}$  often went wrong, with common errors such as  $\frac{1}{3}\mathbf{r}$  and  $-\frac{2}{3}\mathbf{r}$ . Having obtained the correct expression many continued and applied Pythagoras' theorem,  $\sqrt{\left(\frac{1}{2}\mathbf{p}\right)^2 + \left(\frac{1}{3}\mathbf{r}\right)^2}$ . Some gave the answer as a column vector which earned no credit.
  - (iii) A majority of candidates obtained the correct expression, however, just as in **part** (ii), many continued after obtaining a correct expression and applied Pythagoras' theorem or gave the answer as a column vector.
- (b) Many clearly did not know what the term 'position vector' meant. All sorts of combinations of vectors were seen and few of these led to the correct route. Many more might have progressed further if they had shown the point *U* on the diagram. A significant number made no attempt.
- Only a few candidates gained all three marks, partly because magnitude was not clearly understood, even by many of the more able candidates. Common errors for  $|\overline{MT}|$  included 2k.(-k), 2k + (-k) and  $2k^2 \pm k^2$ . These were sometimes equated to 180 but often they were equated to  $\sqrt{180}$ . Many candidates made no attempt at all.

Answers: (a)(i)  $\frac{1}{2}$  p (ii)  $\frac{1}{2}$  p  $-\frac{1}{3}$  r (iii) p +  $\frac{2}{3}$  r (b)  $\frac{3}{2}$  p + r (c) 6

#### **Question 8**

- (a) Most candidates were able to set up the equation and solve it correctly. Some stopped after finding g(1) = 5 and others substituted the 5 into f(x).
- (b) Again, most candidates obtained the correct answer. Some did not understand the process required for the composite function and treated the question as the product of two functions. A few tried to express the composite function in terms of x as a first step but quite often  $2 \times 2^x$  was written as  $4^x$ .
- The process of finding the inverse function was well understood and many correct answers were seen. Some candidates made an error with the signs but usually picked up one of the marks for a correct start. A significant number of candidates treated  $f^{-1}$  as a reciprocal and  $\frac{1}{2x+1}$  was often seen.

- Although many correct answers were seen this proved more challenging and many earned the method mark only. Expanding  $(2x + 1)^2$  proved the downfall with  $2x^2$  often seen instead of  $4x^2$  and  $4x^2 + 1$  another common error. As in **part** (b), some treated gf(x) as a product of the functions and  $(x^2 + 4)(2x + 1)$  was often seen.
- (e) This was a challenging question and only the more able candidates could obtain the correct answer. Few realised that if  $h^{-1}(x) = 0.5$  then h(0.5) = x which leads to  $2^{0.5} = x$ . A variety of errors, such as treating  $h^{-1}(x)$  as a reciprocal, were seen. Two common incorrect responses were 2 and  $\frac{1}{2}$ .
- Few candidates seemed happy to work with powers of 2 and many incorrect responses were seen. Some were able to write  $\frac{1}{h(x)}$  as  $2^{-x}$  but then went wrong in trying to solve -x = kx, sometimes given as k = -2x.

Answers: (a) 2 (b) 17 (c) 
$$\frac{x-1}{2}$$
 (d)  $4x^2 + 4x + 5$  (e)  $\sqrt{2}$  (f)  $-1$ 

# **Question 9**

- (a) A good response was seen with many candidates showing an understanding of the relationship between 'similarity' and 'ratio'. There were, however, quite a number of candidates with answers of 6.5, found by subtracting 7 from 10.5 and then adding this to AB to give their PQ.
- (b) (i) There were few correct answers seen because candidates were expected to write answers specific to the lengths of the sides of the prisms. Many simply stated that the sides were in the same ratio.
  - (ii) Again, very few correct answers were seen. Many appeared to have little understanding of the relationship between lengths and volumes of similar solids. For that reason it was rare to see any mention of ratio, even 2:3. Some answers were given as wrong numerical values, and not in terms of V. Many candidates made no attempt.
- (c) A few candidates used the sine rule correctly, but most simply stated an incorrect angle, seemingly by guesswork. A number of candidates who, correctly, realised that the sine rule was needed, were not able to work this through, making errors in the rearrangement.

Answers: (a) 4.5 (b)(i) 12 and 18 are also in the ratio 2:3 (ii)  $\frac{27V}{8}$  (c) 23.7

# **Question 10**

- (a) (i) A small majority successfully obtained the equation of the line passing through *A* and *B*. For some, errors arose in calculating the gradient, either from errors with signs or from using change in *x* over change in *y*. Once a gradient was calculated, some realised that the intercept was given on the diagram but others attempted a substitution and did not always reach a correct value.
  - (ii) As this was a question requiring candidates to show a particular result it was expected that candidates should use the co-ordinates of A and B to find the values a and b. Many assumed these values and attempted to show the equation balanced. Those who approached it correctly sometimes struggled with the manipulation of the algebraic fractions and lost their way. A few incorrectly substituted x = 4 and y = 2 at the same time.

- (b) (i) Only the more able candidates made any progress with this question. Many did realise the need to substitute the co-ordinates of P or Q into the equation of the curve. Some were successful but after reaching  $\frac{4}{16} + \frac{k^2}{4} = 1$  the elimination of the fractions resulted in many errors. Other candidates forgot to square k. Many others made no attempt.
  - (ii) This trigonometry question proved a challenge to many. The most common incorrect answer was 90°. Those that realised that trigonometry was needed often attempted to calculate *OP* and then use sine. A valid method but all too often accuracy was lost by premature approximation at intermediate stages. Others attempted the cosine rule for triangle *OPQ* but usually lost accuracy for the same reason. Those that attempted the tangent ratio were usually more successful, apart from when candidates forgot to double their answer. As many candidates had an incorrect or no value for *k*, accuracy marks were rarely awarded in this part.
- (c) (i) Although this part was independent of what had gone before many made no attempt. The award of the mark was rare. The most common answer was  $64\pi$ , simply taking a to be 16 and b to be 4. A few did work correctly but either gave the answer as 8 or gave the answer as a decimal.
  - (ii) Very few used an area scale factor to obtain their answer. There was some evidence of using a scale factor of 3 with some of the solutions. A more popular but unsuccessful method was to try and find *b* using *a* as 12. An extremely high number of candidates made no attempt.

Answers: (a)(i)  $-\frac{1}{2}x + 2$  (b)(i)  $\sqrt{3}$  (ii) 81.8 (c)(i)  $8\pi$  (ii)  $72\pi$